5-2 Dividing Monomials

Objective: To simplify quotients of monomials and to find the GCF of several monomials.

Vocabulary

Greatest common factor (GCF) of two or more monomials The common factor with the *greatest coefficient* and the *greatest degree* in each variable. For example, $5x^2y$ is the GCF of $10x^2y^2$ and $25x^3y$.

Properties and Rules

Property of Quotients

If a, b, c, and d are real numbers with $b \neq 0$, and $d \neq 0$, then $\dfrac{ac}{bd} = \dfrac{a}{b} \cdot \dfrac{c}{d}$.

For example, $\dfrac{15}{16} = \dfrac{3 \cdot 5}{2 \cdot 8} = \dfrac{3}{2} \cdot \dfrac{5}{8}$

Rule for Simplifying Fractions

If b, c, and d are real numbers with $b \neq 0$ and $d \neq 0$, then $\dfrac{bc}{bd} = \dfrac{c}{d}$.

For example, $\dfrac{15}{18} = \dfrac{3 \cdot 5}{3 \cdot 6} = \dfrac{5}{6}$.

Rule of Exponents for Division

If a is a nonzero real number and m and n are positive integers, then:

If $m > n$: | If $n > m$: | If $m = n$:
$$\frac{a^m}{a^n} = a^{m-n} \qquad \frac{a^m}{a^n} = \frac{1}{a^{n-m}} \qquad \frac{a^m}{a^n} = 1$$

CAUTION You can divide the numerator and denominator of a fraction only by a nonzero number. In the examples of this lesson, *assume that no denominator equals zero.*

Example 1 Simplify: **a.** $\dfrac{28}{35}$ **b.** $\dfrac{-15xy}{21x}$

Solution **a.** Divide both numerator and denominator by 7. The "cancel" marks show this.

$$\frac{28}{35} = \frac{4 \cdot 7}{5 \cdot 7} = \frac{4}{5}$$

b. Divide both numerator and denominator by $3x$.

$$\frac{-15xy}{21x} = \frac{3x(-5y)}{3x \cdot 7} = \frac{-5y}{7}, \text{ or } -\frac{5y}{7}$$

Example 2 Simplify: **a.** $\dfrac{x^8}{x^3}$ **b.** $\dfrac{x^3}{x^8}$ **c.** $\dfrac{x^2}{x^2}$

Solution **a.** $\dfrac{x^8}{x^3} = x^{8-3} = x^5$ **b.** $\dfrac{x^3}{x^8} = \dfrac{1}{x^{8-3}} = \dfrac{1}{x^5}$ **c.** $\dfrac{x^2}{x^2} = 1$

5-2 Dividing Monomials (continued)

Simplify. Assume that no denominator equals zero.

1. $\dfrac{25}{30}$ 2. $\dfrac{48}{72}$ 3. $\dfrac{54}{72}$ 4. $\dfrac{10^3}{10^6}$ 5. $\dfrac{10^8}{10^5}$ 6. $\dfrac{10a}{2a}$

7. $\dfrac{12m}{4m}$ 8. $\dfrac{15 \cdot 10^3}{5 \cdot 10^4}$ 9. $\dfrac{6x^4}{9x^2}$ 10. $\dfrac{4n^6}{20n^4}$ 11. $\dfrac{2x^5}{16x^4}$ 12. $\dfrac{12y^3}{3xy^2}$

13. $\dfrac{4a^2b}{16ab^2}$ 14. $\dfrac{-6x^2y^3}{9xy^2}$ 15. $\dfrac{-8a^2b}{-20ab}$ 16. $\dfrac{-32cd^3}{-24bd^2}$ 17. $\dfrac{-21bc^3}{-14cd^2}$

18. $\dfrac{30xz^3}{-35yz^2}$ 19. $\dfrac{x^2yz^3}{x^3y^3z^3}$ 20. $\dfrac{a^2b^4c}{a^2bc^3}$ 21. $\dfrac{35a^2b^3c}{25abc}$ 22. $\dfrac{26x^2yz}{52xyz}$

Example 3 $\dfrac{(9x)^2}{(3x)^3} = \dfrac{81x^2}{27x^3} = \dfrac{\cancel{27x^2} \cdot 3}{\cancel{27x^2} \cdot x} = \dfrac{3}{x}$

Simplify. Assume that no denominator equals zero.

23. $\dfrac{(2x)^3}{2x^3}$ 24. $\dfrac{5m^2}{(5m)^2}$ 25. $\dfrac{(2t^2)^3}{(2t^3)^2}$ 26. $\dfrac{(4a^2)^3}{(4a^3)^2}$ 27. $\dfrac{(3ab)^2}{3a^2b}$

28. $\dfrac{(2mn)^3}{2m^3n^2}$ 29. $\dfrac{(-z)^6}{(-z)^3}$ 30. $\dfrac{(-a)^5}{(-a)^3}$ 31. $\dfrac{(-xy)^7}{xy^7}$ 32. $\dfrac{(-t^3)^4}{(-t^2)^5}$

Example 4 Find the missing factor. $45x^2y^3z^4 = (3xyz^2)(?)$ **Solution** $\dfrac{45x^2y^3z^4}{3xyz^2} = 15xy^2z^2$

Find the missing factor.

33. $8t^4 = (2t)(?)$ 34. $10w^4 = (2w^2)(?)$ 35. $6a^3b^5 = (2a^2b^2)(?)$

36. $15pq^3 = (5pq)(?)$ 37. $-28x^2y^4 = (7x^2y)(?)$ 38. $-32a^5b^4 = (-8a)(?)$

Example 5 Find the GCF of $18x^3y$ and $10x^2y^3$.

Solution $\left.\begin{array}{l} 18 = 2 \cdot 3^2 \\ 10 = 2 \cdot 5 \end{array}\right\}$ GCF $= 2$ $\left.\begin{array}{l} \\ \left.\begin{array}{l} x^3y \\ x^2y^3 \end{array}\right\} \text{GCF} = x^2y \end{array}\right\}$ GCF $= 2x^2y$

Find the GCF.

39. $21x^3,\ 14x^2$

40. $a^3b^2,\ a^2b^3$

41. $6xy^2,\ 8x^4y^3$

42. $18c^2d^3,\ 24c^2d$

43. $35pq^2r,\ 25p^3qr^2$

Mixed Review Exercises

Simplify.

1. $\dfrac{1}{4}(-24)$ 2. $105 \cdot \dfrac{1}{5}$ 3. $378 \div 9$ 4. $4n^3\left(\dfrac{1}{4}n^3\right)$ 5. $12 \div \left(-\dfrac{1}{3}\right)$ 6. $10y \cdot \dfrac{2}{5}y^2$